



# An Interactive Excel Program for Tracking a Single Droplet in Crossflow Computation

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## Acknowledgments

Undertaking this project has given me (Egel Urip) an enormous knowledge in spray dynamics, and it has given me a chance to employ my skills in solving problems of spray in a Crossflow. I am very grateful for being given this project. I would like take this opportunity to thank my graduate advisor, Dr. S.L. (Jason) Yang, for all his support in solving differential equations; Dr. Cecil J. Marek for his helpful and useful comments. I was truly honored to have worked with these experienced and friendly people. I hope this code will help engineers in exploring the spray in crossflow application. I also hope that it will provide an example for using Excel for other problems. The authors would like to thank Dr. Paul Penko for being the grant monitor on this project; and Dr. William F. Ford of the NASA Glenn Computer Services Division for helping with many details of the code such as using emergency halts, simplifying the code, and controlling the sheet size.

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# **An Interactive Excel Program for Tracking a Single Droplet in Crossflow Computation**

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## **Abstract**

Spray jet in crossflow has been a subject of research because of its wide application in systems involving pollutant dispersion, jet mixing in the dilution zone of combustors, and fuel injection strategies. The focus of this work is to investigate dispersion of a 2-dimensional atomized spray jet into a 2-dimensional crossflow. A quick computational method is developed using available software. The spreadsheet can be used for any 2D droplet trajectory problem where the drop is injected into the free stream eventually coming to the free stream conditions.

During the transverse injection of a spray into high velocity airflow, the droplets (carried along and deflected by a gaseous stream of co-flowing air) are subjected to forces that affect their motion in the flow field. Based on the Newton's Second Law of motion, four ordinary differential equations were used. These equations were then solved by a 4<sup>th</sup>-order Runge-Kutta method using Excel software.

Visual basic programming and Excel macrocode to produce the data facilitate Excel software to plot graphs describing the droplet's motion in the flow field. This program computes and plots the data sequentially without forcing users to open other types of plotting programs. A user's manual on how to use the program is also included in this report.

## **Nomenclature**

$A_d$      Projected area of the droplet,  $\pi r^2$   
 $C_D$      Drag coefficient, *see equation (8)*  
 $\vec{F}$      Force

$g$	Gravitational acceleration constant, $9.8m/s^2$
$n$	$n^{\text{th}}$ iteration
$Re$	Reynolds number, <i>see equation (9)</i>
$r$	Spherical radius of the droplet
$u_g$	Velocity of the cross stream (air) in the x-direction
$u_d$	Velocity of the droplet in the x-direction
$\vec{U}$	Velocity
$\vec{U}_R$	Relative velocity between the droplet and the gas stream
	$\vec{U}_R = (u_d - u_g)\hat{i} + (w_d - w_g)\hat{k}$
$V_d$	Droplet Volume, $\frac{4}{3}\pi r^3$
$w_g$	Velocity of the cross stream (air) in the z-direction
$w_d$	Velocity of the droplet in the z-direction
$x$	horizontal direction $\pm$ equals to the right
$z$	vertical direction $\pm$ equals up
$\rho_g$	Density of the cross stream (air)
$\rho_d$	Density of the droplet
$\mu_g$	Viscosity constant of the gaseous fluid
$\Delta t$	time step-size

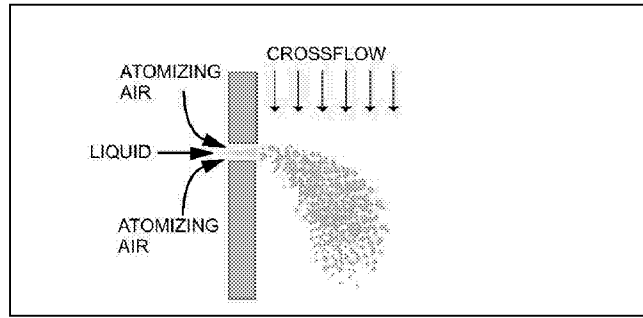
#### SUBSCRIPTS:

$d$	Droplet
$g$	Gaseous phase

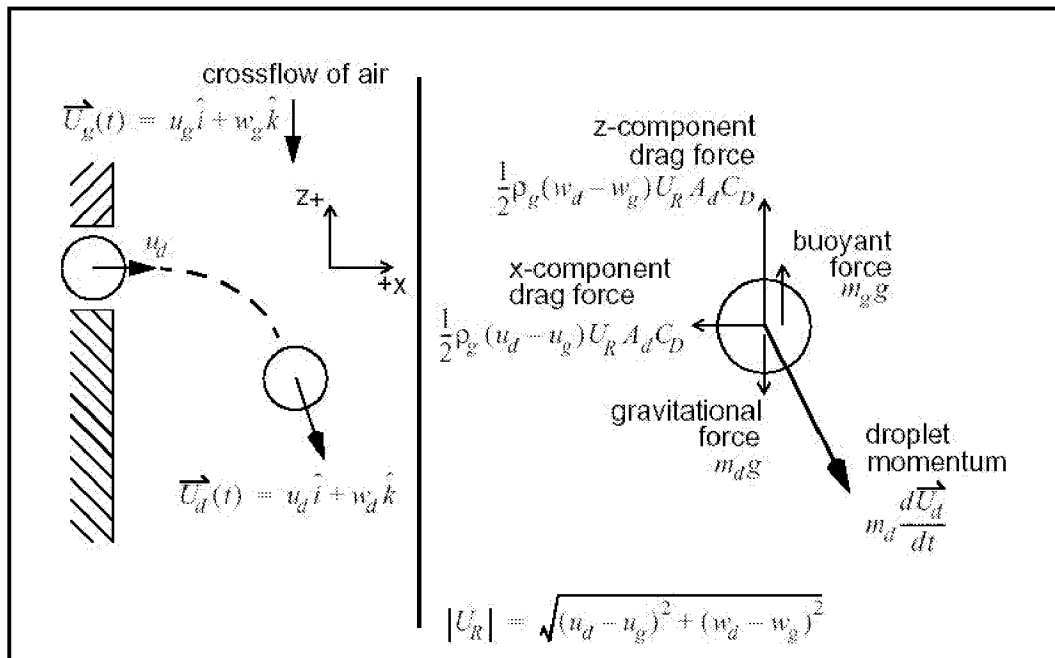
## Introduction and Governing Equations

A liquid spray injected into a gaseous crossflow is important because of its wide application in systems involving two phase mixing and in combustion requiring quick mixing and reduction of pollutants, for jet mixing in the dilution zone of combustors, and for determining fuel injection strategies. It is important to be able to compute this flow to optimize the mixing strategy.

This work is mainly focused on producing a quick computational method for determining spray penetration. With this spreadsheet, one can investigate the dispersion of an air-blast atomized spray jet into a crossflow, see Figure 1. During the transverse injection of a spray into high velocity airflow, the droplets (carried along in the gaseous stream of co-flowing air) are subjected to forces that affect their motion in the flow field (see Figure 2).



**Figure 1 (from NASA/CR—2000-210467, reference 1)**



**Figure 2 (from NASA/CR—2000-210467)**

The trajectories of the droplets can be tracked by applying a Lagrangian-based analysis to the droplets. Since no evaporation is assumed, the code can be used for solid particles as well. The momentum equations for a droplet can be obtained by equating the droplet motion to: (1) The viscosity and pressure-related drag forces, (2) The pressure gradient and viscous forces related to the fluid surrounding the droplet, (3) The inertia of the virtual mass, induced when the particle acceleration affects the fluid mass acceleration, and (4) The Basset force, which takes into account the acceleration history of the droplet.

Based on these principles along with the following assumptions:

- (1) The droplets are spherical,
- (2) No droplets breakup occurs,

- (3) Vaporization is not considered and is assumed negligible, and
- (4) Lift, virtual mass, and Basset forces are neglected.
- (5) Chemical reaction is not included,

droplet trajectory and velocity with respect to time can be calculated. These assumptions reduce the droplet momentum equation to include only the effects of the drag and body forces. The general momentum equations for a single droplet injected along the positive x-direction, transversely into a downward-flowing air stream in the positive z-direction, as shown in Figure 2, is described by

$$\vec{F}_d = \vec{F}_{drag} + \vec{F}_{body} \quad (1)$$

where the net force  $\vec{F}_d$  that drives the droplet motion is balanced by the drag force opposing its motion, and the field forces acting on the droplet. The aerodynamic drag force is given by

$$\vec{F}_{drag} = -\frac{1}{2} \rho_g \vec{U}_R |\vec{U}_R| A_d C_D \quad (2)$$

where  $\rho_g$  is the air density, and  $A_d$  and  $C_D$ , the projected area and the drag coefficient of the droplet, respectively. The relative velocity between the droplet and the crossflow has a magnitude of  $U_R$  (see Figure 2). The subscript “d” refers to the droplet and “g” the crossflow air.

The body force, resulting from an equivalent volume of air that buoys the droplet, includes the gravitational and buoyancy forces. It is given by

$$\vec{F}_{body} = (\rho_g - \rho_d) V_d \vec{g} \quad (3)$$

which says that the body force is equal to the product of relative droplet and air density ( $\rho_d - \rho_g$ ), the droplet volume  $V_d$ , and the gravitational acceleration  $g$ .

Substituting equation 2 and equation 3 to equation 1 yields:

$$\rho_d V_d \frac{du_d}{dt} = -\frac{1}{2} \rho_g (u_d - u_g) |\vec{U}_R| A_d C_D \quad (4)$$

$$\rho_d V_d \frac{dw_d}{dt} = -\frac{1}{2} \rho_g (w_d - w_g) |\vec{U}_R| A_d C_D + (\rho_g - \rho_d) V_d g \quad (5)$$

$$\frac{dx}{dt} = u_d \quad (6)$$



$$\frac{dz}{dt} = w_d \quad (7)$$

The drag coefficient of the droplet depends on the droplet Reynolds number and is given by

$$C_D = \begin{cases} \frac{24}{\text{Re}_d} \left[ 1 + \frac{1}{6} \text{Re}_d^{2/3} \right] & \text{Re}_d \leq 1000 \\ 0.424 & \text{Re}_d > 1000 \end{cases} \quad (8)$$

where  $\text{Re}_d$  is the droplet Reynolds number and is defined as follows

$$\text{Re}_d = \frac{2\rho_g |\vec{U}_R| r_d}{\mu_g} \quad (9)$$

in which  $r_d$  is the droplet radius and  $\mu_g$  is the gas (air) viscosity.

## Numerical Method

Four ordinary differential equations are to be solved, namely, equations 4, 5, 6 and 7, for the four dependent variables  $u_d$ ,  $w_d$ ,  $x$  and  $z$ . The droplet trajectory is defined by the set of  $x$  and  $z$  values. A 4<sup>th</sup>-Order Runge-Kutta explicit method<sup>1</sup> was used to solve these equations. The Runge-Kutta explicit method is an ideal numerical scheme for solving ordinary differential equations using Excel software. It is a self-starting method with good stability characteristic. The step-size can be changed as desired without any complications for higher-order schemes. For a set of two coupled equations, such as,

$$\frac{dx}{dt} = f(x, z, t) \quad (10a)$$

$$\frac{dz}{dt} = g(x, z, t) \quad (10b)$$

the 4<sup>th</sup>-order Runge-Kutta method reads (subscript  $n$  stands for the  $n^{\text{th}}$  time step);  $k$  and  $l$  are unknown constants.

---

<sup>1</sup> Nakamura, Shoichiro, *Applied Numerical Method With Software*, Englewood Cliffs, New Jersey: Prentice Hal, 1991.

$$x_{n+1} = x_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (11a)$$

$$z_{n+1} = z_n + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \quad (11b)$$

where

$$\begin{aligned} k_1 &= \Delta t \cdot f(x_n, z_n, t_n) \\ k_2 &= \Delta t \cdot f\left(x_n + \frac{k_1}{2}, z_n + \frac{l_1}{2}, t_n + \frac{\Delta t}{2}\right) \\ k_3 &= \Delta t \cdot f\left(x_n + \frac{k_2}{2}, z_n + \frac{l_2}{2}, t_n + \frac{\Delta t}{2}\right) \\ k_4 &= \Delta t \cdot f(x_n + k_3, z_n + l_3, t_n + \Delta t) \end{aligned} \quad (12a)$$

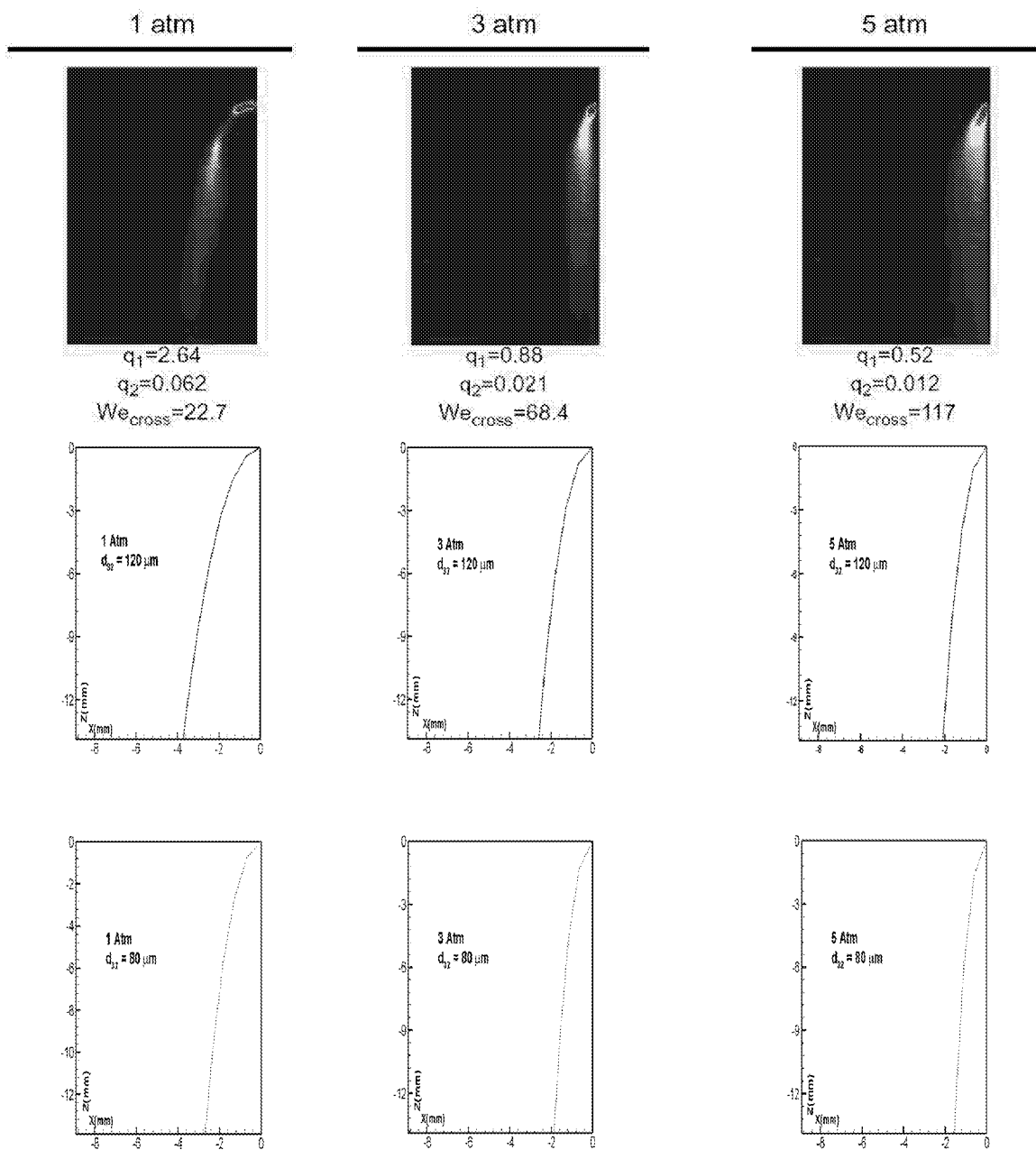
and

$$\begin{aligned} l_1 &= \Delta t \cdot g(x_n, z_n, t_n) \\ l_2 &= \Delta t \cdot g\left(x_n + \frac{k_1}{2}, z_n + \frac{l_1}{2}, t_n + \frac{\Delta t}{2}\right) \\ l_3 &= \Delta t \cdot g\left(x_n + \frac{k_2}{2}, z_n + \frac{l_2}{2}, t_n + \frac{\Delta t}{2}\right) \\ l_4 &= \Delta t \cdot g(x_n + k_3, z_n + l_3, t_n + \Delta t) \end{aligned} \quad (12b)$$

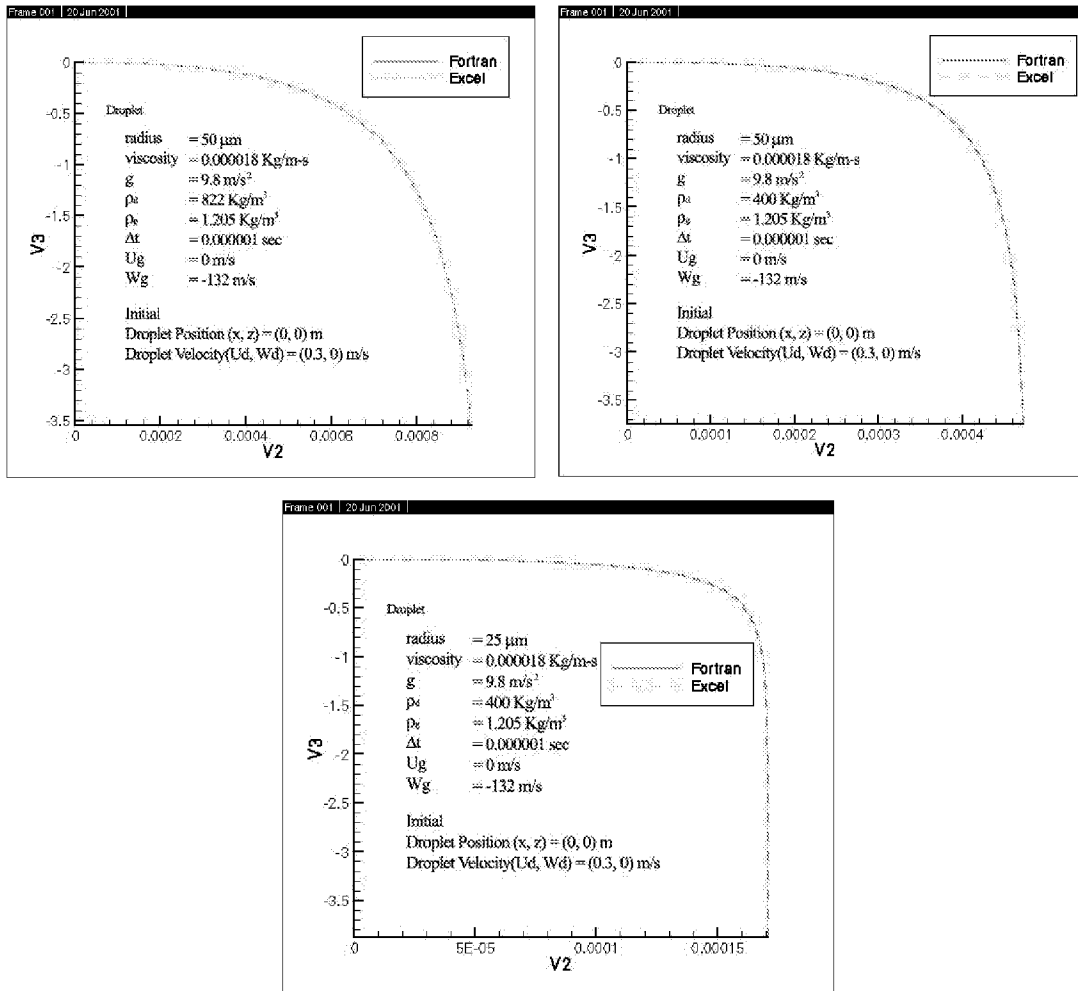
Only every nth cycle (as specified by the user) is saved for plotting. This greatly saves on storage and increases the speed of post processing. We have chosen to enter the data in SI units in the unlocked cells. The required conversions are done in the locked cells. When the user becomes familiar with the spreadsheet, the spreadsheet can be unlocked, with password NASA, and the user can adapt the spreadsheet as required.

The predictions were compared with the data of reference 1 with good results, see Figures 3 and 4. The experimental data from Figure 9.4 in ref. 1 for cross flow jets without an air blast assist was used. Jet to crossflow momentum flux ratio is used in this study to determine the depth of the droplet penetration. Momentum flux ratio for single phase jet is given by

$$q_1 = \frac{\rho U^2|_{jet}}{\rho U^2|_{crossflow}},$$



**Figure 3—Experimental Comparison**



**Figure 4—Droplet Trajectory Data Validation**

and momentum flux ratio for two phase jet is given by

$$q_2 = \frac{(\rho_L U_L^2 A_{fuel} + \rho_g U_{airbl}^2 A_{airbl})}{\rho_g U_{cross}^2} / A_{spray}$$

The 120 micron droplet predicted the most penetration and the 80 micron droplet predicted the least penetration.

To validate the results obtained using the Excel spreadsheet (because no analytical solution is available), a FORTRAN program was also developed for this purpose and is given in the appendix along with three validation cases (plots). FORTRAN and Excel calculations are compared in Figure 5. With an Excel spreadsheet, we do not have to compile, build, and link as in a regular FORTRAN code. In addition, the graphics are immediately displayed after the computations are completed, so that the results are seen quickly and changes in the input can be made. The interactive spreadsheet is available on this CD as a separate document. Additional copies of the spreadsheet can be requested by e-mailing: [Cecil.J.Marek@grc.nasa.gov](mailto:Cecil.J.Marek@grc.nasa.gov). The report portion can be accessed on the web at: <http://gltrs.grc.nasa.gov/cgi-bin/GLTRS/browse.pl?2002/TM-2002-211710.html>

## User's Manual

This program is written in Microsoft Visual Basic Excel. There are three sheets in the program, namely the Instruction Sheet, the Process Sheet, and the Code Sheet.

### Instructions Sheet

The instruction sheet contains a brief description of the problem, the solution method, and the user-input variables. Several schematics, which describe the forces applied on the droplet, can also be found on this sheet.

### Process Sheet

The process sheet contains the user-inputs and the solution plots. There are five graphs on this sheet (see Figure 6), namely droplet trajectory, droplet velocity profile, drag coefficient,  $C_D$ , as a function of time, droplet velocity profiles as a function of time, and droplet trajectory profiles as a function of time. The cells highlighted in green are the user inputs. The **cells highlighted in cyan** contain computed values associated with the green cells; therefore, they are locked to prevent the user from modifying the values. When values are entered into the formula cells, the formulas are erased and linkages to other cells are interrupted. That is why for this version we chose to lock the closed cells without a password.

Input Sheet			
<b>Inputs</b>			
Spherical radius of the droplet ( $r_{s2}$ )	$r_{s2} =$	60 $\mu\text{m}$	or 0.00006 m
Viscosity of the crossflow fluid	$\mu_0 =$	2.00E-05 Kg/m-s	
Gravitational Acceleration	$g =$	9.8 m/s <sup>2</sup>	
Droplet Density	$\rho_d =$	777 Kg/m <sup>3</sup>	
Fluid Density	$\rho_0 =$	1.22 Kg/m <sup>3</sup>	
Time Step	$\Delta t =$	1.00E-06 sec	
Fluid Velocity (X direction)	$U_0 =$	0 m/s	
Fluid Velocity (Z direction)	$W_0 =$	-38 m/s	
<b>Variable Descriptions</b>			
Initial Droplet Position (X direction)	$X_0 =$	0 m	
Initial Droplet Velocity (X direction)	$U_{d0} =$	-2.4 m/s	
Initial Droplet Position (Z direction)	$Z_0 =$	0 m	
Initial Droplet Velocity (Z direction)	$W_{d0} =$	0 m/s	
Projected area of the droplet	$A_d =$	1.1314E-08 m <sup>2</sup>	or 1314.29 $\mu\text{m}^2$
Volume of the droplet	$V_d =$	9.0514E-13 m <sup>3</sup>	or 905142.9 $\mu\text{m}^3$
Total Cycles (INTEGER NUMBER)		60000 Cycles	
Data taken every (INTEGER NUMBER)		300 Cycles	
Data plot for		6.00E-01 seconds, 2001	Data Written
Coordinate Systems	X-RIGHT	POSITIVE	
	Z-UP	POSITIVE	
<b>COMMAND</b>		Update	Halt
<b>COMPLETION</b>			

Figure 5

Input Sheet

Inputs  
Spherical radius of the droplet ( $r_{12}$ )  $r_{12} =$  60 m or 0.00908 m  
Viscosity of the crossflow fluid  $\mu =$  2.00E-05 Kg/m-s  
Gravitational Acceleration  $g =$  9.8 m/s<sup>2</sup>  
Droplet Density  $\rho_d =$  822 Kg/m<sup>3</sup>  
Fluid Density  $\rho_f =$  1.22 Kg/m<sup>3</sup>  
Time Step  $t =$  1.00E-06 sec  
Fluid Velocity (X direction)  $U_g =$  0 m/s  
Fluid Velocity (Z direction)  $W_g =$  -38 m/s

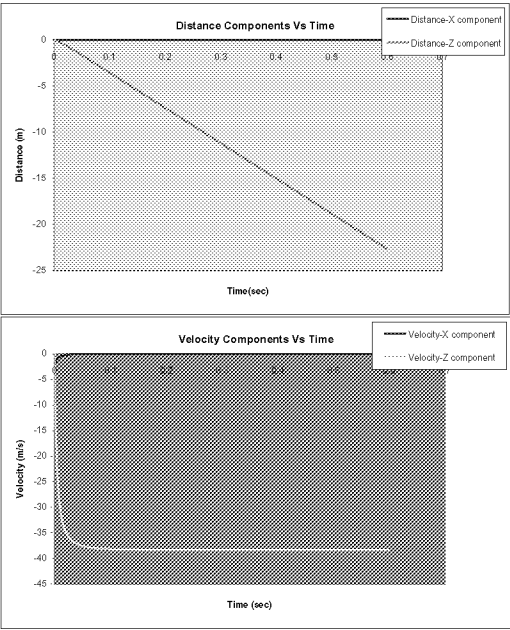
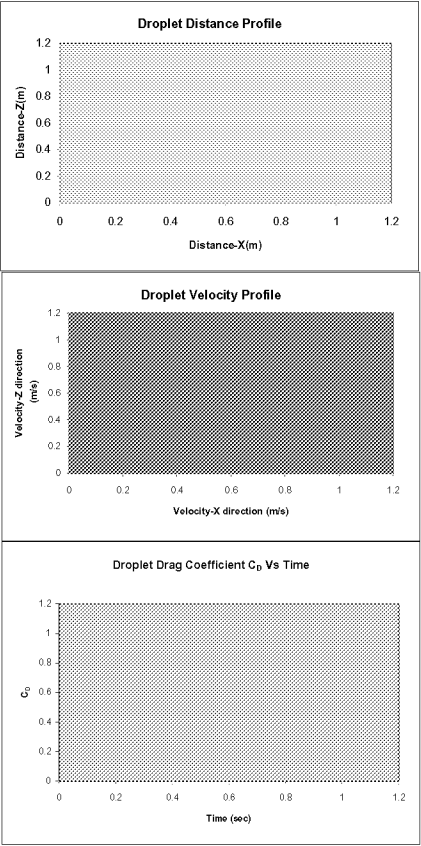
Initial Droplet Position (X direction)  $X_o =$  0 m  
Initial Droplet Velocity (X direction)  $U_{d_o} =$  -2.4 m/s  
Initial Droplet Position (Z direction)  $Z_o =$  0 m  
Initial Droplet Velocity (Z direction)  $W_{d_o} =$  0 m/s

Projected area of the droplet  $A_d =$  1.13143E-08 m<sup>2</sup> or 11314.29 m<sup>2</sup>  
Volume of the droplet  $V_d =$  9.05143E-13 m<sup>3</sup> or 905142.9 m<sup>3</sup>  
Total Cycles (INTEGER NUMBER) 600000 Cycles  
Data taken every (INTEGER NUMBER) 300 Cycles  
Data plot for 6.00E-01 seconds; 2001 Data Written

Coordinate Systems X—RIGHT POSITIVE  
Z—UP POSITIVE

COMMAND	<input type="button" value="OK"/>	<input type="button" value="Exit"/>	<input type="button" value="Clear"/>
COMPLETION	<input type="button" value="OK"/>		

OPTIONAL: For TecPlot Viewer ONLY  
Do you want to save the data into a file? ☒ Yes  
☐ No



From Figure 5, the last two user inputs, *Total Cycles (C25)* and *Data taken every ### cycles (C26)* are included. *The number assigned in cyan cell “E27” must be kept below 65,536; the cell will turn into red if this condition is not satisfied.* Cell E27 basically shows amount of data will be written into the code sheet, and Excel can only hold 65,536 rows of data. Keeping the value below the limit can be done by changing the value in the cell “C26”.

After all the inputs have been specified, clicking the *Update* button will instruct the program to update the data and the five plots.

In summary, the user needs to do the following steps to run the program:

1. Go to the Process Sheet **E** by clicking on the *Process* tab
2. Enter input values in the green cells
3. Adjust the value in the cell C26 so that the computed value in cell E27 is less than 65,536
4. Click the **Update** button
5. Observe the droplet profiles on the five solution plots
6. Repeat step 1 through step 5 for different input values
7. Click the **Clear Data** button to clear the data (Optional).

Additional features include the option to store the computed data into a TecPlot format file. This feature provides the user a flexibility to plot the data using other software such as TecPlot. The details about how to run this option are explained in the discussion section below.

### Code Sheet

The last sheet is the code sheet, which contains the solution data produced by the program. Six columns namely A, B, C, D, E, and F hold the data of time, trajectory x-component, trajectory z-component, velocity x-component, velocity z-component, and drag coefficient respectively.

### Discussions

The five plots are based on the X-Z coordinate system where X positive is to the right and Z positive is up. If the inputs of the droplet’s velocity injection is set only in the X positive component, and the crossflow’s velocities are set to zero, you will see a decreasing slope profile on the trajectory plot. Because relative velocity in the Z direction between the droplet and the crossflow is zero, gravity force overcomes zero buoyancy force, and results in the droplet to move down, in the negative Z direction.

The total number of cycles will determine the time for the program to process the data. *Data taken every cycle* also determines the speed of the program to process the data. The more data is collected, the slower the program to complete the cycle.



You are encouraged to change the chart type of the five plots which you find best and easy to analyze. You might want to switch to *dotted points-connected line chart* to analyze the data points.

The cells highlighted in cyan and in white are locked for the coding security purpose. It is highly recommended that initially the user not *unlock and modify* these cells. Any modifications made may cause the program to crash.

The last feature added to this program is the flexibility for the user to store the computed data into TecPlot format file. By enabling this option, the user needs to specify the path and the filename to store the output. This option must be *disabled* if the path and the file name are not specified.



## Appendix A

### Visual Basic Code

Under the 'Tools', 'Macro', 'Visual Basic' button, the heart of the numerical code is presented. This code is reproduced here in case something happens to the program. Its logic is similar to the FORTRAN code following, but some things are different.

```
Private Sub CommandButton3_Click()  
    Dim n As Long, nn As Double, nnnn As Long, userchoice As Long  
    Dim xx(0 To 1) As Double, xp(0 To 1) As Double  
    Dim zz(0 To 1) As Double, zp(0 To 1) As Double  
    Dim Cdm As Double, Rems As Double  
    Dim A_x As Double, A_z As Double, h As Double, kl As Double, r As Double, ht As  
Double  
    Dim k1 As Double, k2 As Double, k3 As Double, k4 As Double, l1 As Double, l2 As  
Double, l3 As Double, l4 As Double  
    Dim kz1 As Double, kz2 As Double, kz3 As Double, kz4 As Double, lz1 As Double,  
lz2 As Double, lz3 As Double, lz4 As Double  
    Dim Urm As Double  
    Dim ug As Double, wg As Double, mu As Double, rg As Double  
    Dim a As Double, b As Double  
    Dim location  
    Dim sd, sf  
    Dim a1, b1, c1, d1, e1, f1  
    Dim unitconv  
  
    Call Macro2  
    Halt = False  
    Msg = "Do you want to continue ?"  
    Style = vbYesNo  
    Title = "Jet Flow in CrossFlow"  
  
    'Getting the input  
    ug = Sheets("Process").Cells(11, 3)  
    wg = Sheets("Process").Cells(12, 3)  
    mu = Sheets("Process").Cells(6, 3)  
    rg = Sheets("Process").Cells(9, 3)  
    rd = Sheets("Process").Cells(8, 3)  
    g = Sheets("Process").Cells(7, 3)  
    xx(0) = Sheets("Process").Cells(17, 3)  
    xp(0) = Sheets("Process").Cells(18, 3)  
    zz(0) = Sheets("Process").Cells(19, 3)  
    zp(0) = Sheets("Process").Cells(20, 3)  
    h = Sheets("Process").Cells(10, 3)
```

```

k1 = Sheets("Process").Cells(25, 3)
userchoice = Sheets("Process").Cells(26, 3)
r = Sheets("Process").Cells(5, 3)
r = r / 1000000#

```

```

Worksheets("Process").CommandButton1.Width = 0
Worksheets("Process").CommandButton1.Visible = True

```

```

n = 0
nn = -1
nnn = 1
ht = 0

```

20

```

ht = ht + h
a = xp(n)
b = zp(n)
Call Runge(a, b, r, rd, rg, g, mu, ug, wg, A_x, A_z)
If (SuperExit) Then GoTo 50
k1 = h * xp(n)
l1 = h * A_x
kz1 = h * zp(n)
lz1 = h * A_z

```

```

a = xp(n) + l1 / 2#
b = zp(n) + lz1 / 2#
Call Runge(a, b, r, rd, rg, g, mu, ug, wg, A_x, A_z)
If (SuperExit) Then GoTo 50
k2 = h * xp(n)
l2 = h * A_x
kz2 = h * zp(n)
lz2 = h * A_z

```

```

a = xp(n) + l2 / 2#
b = zp(n) + lz2 / 2#
Call Runge(a, b, r, rd, rg, g, mu, ug, wg, A_x, A_z)
If (SuperExit) Then GoTo 50
k3 = h * xp(n)
l3 = h * A_x
kz3 = h * zp(n)
lz3 = h * A_z

```

```

a = xp(n) + l3
b = zp(n) + lz3
Call Runge(a, b, r, rd, rg, g, mu, ug, wg, A_x, A_z)

```

If (SuperExit) Then GoTo 50

k4 = h \* xp(n)

l4 = h \* A\_x

kz4 = h \* zp(n)

lz4 = h \* A\_z

xp(n + 1) = xp(n) + (1# / 6#) \* (l1 + 2# \* l2 + 2# \* l3 + l4)

zp(n + 1) = zp(n) + (1# / 6#) \* (lz1 + 2# \* lz2 + 2# \* lz3 + lz4)

xx(n + 1) = xx(n) + (1# / 6#) \* (k1 + 2# \* k2 + 2# \* k3 + k4)

zz(n + 1) = zz(n) + (1# / 6#) \* (kz1 + 2# \* kz2 + 2# \* kz3 + kz4)

nn = nn + 1#

If nn = 0 Then

Urm = (((xp(n) - ug) ^ 2) + ((zp(n) - wg) ^ 2)) ^ 0.5

Rems = 2 \* rg \* Urm \* r / mu

If Rems <= 1000 Then

Cdm = (((Rems ^ (2 / 3)) / 6) + 1) \* 24 / Rems

Else

Cdm = 0.424

End If

Sheets("Code").Cells(2, 1) = nn \* h

Sheets("Code").Cells(2, 2) = xx(n)

Sheets("Code").Cells(2, 3) = zz(n)

Sheets("Code").Cells(2, 4) = xp(n)

Sheets("Code").Cells(2, 5) = zp(n)

Sheets("Code").Cells(2, 6) = Cdm

Worksheets("Process").CommandButton1.Width = ((nn / kl) \* 165.75)

Worksheets("Process").CommandButton1.Caption = ((nn / kl) \* 100) & "%"

Worksheets("Process").CommandButton1.Height = 20.25

Else

If 0 = (nn Mod userchoice) Then

nnn = nnn + 1

Urm = (((xp(n + 1) - ug) ^ 2) + ((zp(n + 1) - wg) ^ 2)) ^ 0.5

Rems = 2 \* rg \* Urm \* r / mu

If Rems <= 1000 Then

Cdm = (((Rems ^ (2 / 3)) / 6) + 1) \* 24 / Rems

Else

Cdm = 0.424

End If

Sheets("Code").Cells(nnn + 1, 1) = nn \* h

Sheets("Code").Cells(nnn + 1, 2) = xx(n + 1)

Sheets("Code").Cells(nnn + 1, 3) = zz(n + 1)

Sheets("Code").Cells(nnn + 1, 4) = xp(n + 1)

```

Sheets("Code").Cells(nnn + 1, 5) = zp(n + 1)
Sheets("Code").Cells(nnn + 1, 6) = Cdm
Worksheets("Process").CommandButton1.Width = ((nn / kl) * 165.75)
Worksheets("Process").CommandButton1.Caption = ((nn / kl) * 100) & "%"
Worksheets("Process").CommandButton1.Height = 20.25

```

```

DoEvents
If (Halt) Then
    DoEvents
    Halt = False
    Response = MsgBox(Msg, Style, Title)
    If Response = vbNo Then
        GoTo 10
    Else
        End If
    End If
End If

```

```

Else
End If
End If

```

```

xp(n) = xp(n + 1)
zp(n) = zp(n + 1)
xx(n) = xx(n + 1)
zz(n) = zz(n + 1)

```

```

If nn >= kl Then
    GoTo 10
Else
    GoTo 20
End If

```

```

10
Macro1
sd = Sheets("Process").Cells(27, 5)
If Optional_Save = "YES" Then
    location = Sheets("Process").Cells(44, 2)
    Open location For Output As #1
    Print #1, "TITLE = "; Spc(2); "*****"; "Spray Jet In CrossFlow"; "*****"
    If Unit = "millimeter" Then
        unitconv = 1000#
        Print #1, "Variables ="; Spc(2); "*****"; "Time(sec)"; "*****"; Spc(2); "*****"; "X(mm)"; "*****";
        Spc(2); "*****"; "Z(mm)"; "*****"; Spc(2); "*****"; "U(mm/s)"; "*****"; Spc(2); "*****"; "W(mm/s)";
        "*****"; Spc(2); "*****"; "Cd"; "*****"
    Else

```

```

    unitconv = 1#
    Print #1, "Variables ="; Spc(2); "Time(sec)"; "X(meter)"; "Z(meter)"; "U(m/s)"; "W(m/s)"; "Cd";
    End If
    Print #1, "ZONE I ="; sd & ", "; Spc(2); "F = POINT"
    For sf = 2 To sd + 1
        a1 = Sheets("Code").Cells(sf, 1)
        b1 = Sheets("Code").Cells(sf, 2)
        b1 = b1 * unitconv
        c1 = Sheets("Code").Cells(sf, 3)
        c1 = c1 * unitconv
        d1 = Sheets("Code").Cells(sf, 4)
        d1 = d1 * unitconv
        e1 = Sheets("Code").Cells(sf, 5)
        e1 = e1 * unitconv
        f1 = Sheets("Code").Cells(sf, 6)
        Print #1, a1; Spc(2); b1; Spc(2); c1; Spc(2); d1; Spc(2); e1; Spc(2); f1
    Next sf
    Close #1
Else
End If
50
SuperExit = False
End Sub

```

```

Sub Runge(xps, zps, rs, rds, rgs, g, mus, ugs, wgs, A_xs, A_zs)
    Dim Area As Double, Volume As Double, Ur As Double, Re As Double, Cd As Double

    Area = (22# / 7#) * rs ^ 2
    Volume = (22# / 7#) * (4# / 3#) * rs ^ 3

    Ur = (((xps - ugs) ^ 2#) + ((zps - wgs) ^ 2#)) ^ 0.5
    Msg = "The Time Step Size is too big. Program Terminated Ur = " & Ur
    Msg2 = "Reduce the time step size!"
    Style = vbOKOnly
    Title = "Jet Flow in CrossFlow"

    If Ur > (10 ^ 80) Then
        Response = MsgBox(Msg, Style, Title)
        Response = MsgBox(Msg2, Style, Title)
        SuperExit = True
        Sheets("Process").Range("C10").Select
    Else: End If
    Re = 2# * rgs * Ur * rs / mus

```

```

If Re <= 1000 Then
    Cd = Re ^ (2# / 3#)
    Cd = Cd / 6#
    Cd = (Cd + 1#) * 24# / Re
Else
    Cd = 0.424
End If
A_xs = -0.5 * rgs * Ur * Area * Cd * (xps - ugs) / (rds * Volume)
A_zs = (-0.5 * rgs * Ur * Area * Cd * (zps - wgs) + (rgs - rds) * Volume * g) / (rds *
Volume)
End Sub

```



## Appendix B

### Equivalent Fortran Program

```

*deck main
C=====
C
C
C This program calculates droplet's trajectory and its velocity *
C after it is injected into a cross-stream using the 4th-order *
C Runge-Kutta method. It calls derivs which defines the ordinary *
C differential equations (ODEs). *
C All the constant values can be found in subroutine derivs *
C=====
C
C      parameter (nd=10000000)
C      dimension x(nd),dx(nd),u(nd),du(nd),t(nd),xt(nd),ut(nd),
C      &          dxt(nd),dut(nd),dxm(nd),dum(nd)
C      dimension z(nd),dz(nd),w(nd),dw(nd),zt(nd),wt(nd),
C      &          dzt(nd),dwt(nd),dzm(nd),dwm(nd)
C
C +++
C +++ input parameters
C +++
C      print*,'Enter time step : '
C      read*,dt
C      print*,' '
C      print*,'Enter total time: '
C      read*,tend
C      print*,'Enter initial droplet position x(0) : '
C      read*,x(1)
C      print*,' '
C      print*,'Enter initial droplet velocity u(0) : '
C      read*,u(1)
C      print*,' '
C      print*,'Enter initial droplet position z(0) : '
C      read*,z(1)
C      print*,' '
C      print*,'Enter initial droplet velocity w(0) : '
C      read*,w(1)
C      print*,' '
C      print*,'Enter radius [micro-meter]: '
C      read*,r
C      print*,' '
C      r=r*1.e-6
C
C      pi=acos(-1.)
C      ad=pi*r**2      ! Droplet area
C      vd=4./3.*pi*r**3 ! Droplet volumn
C
C      print*,'Area = ',ad
C      print*,'Volumn = ',vd
C
C      t(1)=0.0
C
C      n=tend/dt
C

```

```

write(*,*)
write(*,*)' time step dt = ',dt
write(*,*)

if(n.gt.nd)then
  write(*,*)' n is greater than nd -- run aborting'
  write(*,*)' n = ',n,' nd = ',nd
  stop
endif
C
  t(n)=tend
C +++
C +++ Solving the differential equations using 4th-order
C +++ Runge Kutta methods.
C +++
  dtt=0.5*dt
  do 10 i=1,n-1
C +++
C +++
    call derivs(i,r,ad,vd,u,dx,du,w,dz,dw)
C +++
C +++
    xt(i)=x(i)+dtt*dx(i)
    ut(i)=u(i)+dtt*du(i)
    zt(i)=z(i)+dtt*dz(i)
    wt(i)=w(i)+dtt*dw(i)
    call derivs(i,r,ad,vd,ut,dxt,dut,wt,dzt,dwt)
C +++
C +++
    xt(i)=x(i)+dtt*dxt(i)
    ut(i)=u(i)+dtt*dut(i)
    zt(i)=z(i)+dtt*dzt(i)
    wt(i)=w(i)+dtt*dwt(i)
    call derivs(i,r,ad,vd,ut,dxm,dum,wt,dzm,dwm)
C +++
C +++
    xt(i)=x(i)+dt*dxm(i)
    ut(i)=u(i)+dt*dum(i)
    dxm(i)=dxt(i)+dxm(i)
    dum(i)=dut(i)+dum(i)
    zt(i)=z(i)+dt*dzm(i)
    wt(i)=w(i)+dt*dwm(i)
    dzm(i)=dzt(i)+dzm(i)
    dwm(i)=dwt(i)+dwm(i)
    call derivs(i,r,ad,vd,ut,dxt,dut,wt,dzt,dwt)
C +++
C +++ advance the solutions to n+1 time step
C +++
    t(i+1)=t(i)+dt
    x(i+1)=x(i)+dt/6.*(dx(i)+dxt(i)+2.*dxm(i))
    u(i+1)=u(i)+dt/6.*(du(i)+dut(i)+2.*dum(i))
    z(i+1)=z(i)+dt/6.*(dz(i)+dzt(i)+2.*dzm(i))
    w(i+1)=w(i)+dt/6.*(dw(i)+dwt(i)+2.*dwm(i))
C
  10 continue
C
C +++

```

```

c +++ write data sets to file result.dat
c +++
  open(unit=3,file='result.dat',form='formatted')
  do 20 i=1,n
    write(3,100) t(i),x(i),z(i),u(i),w(i)
  20 continue
c
  write(*,*)' The data is written to result.dat as'
  write(*,*)' t, x, z, u, w'
c
  close(3)
c
  100 format(1x,f5.3,4(1x,f15.10))
  end
c
c=====
*deck derivs
  subroutine derivs(i,r,ad,vd,u,dx,du,w,dz,dw)
c -----*
c This subroutine defines the differential equations *
c being solved by the program. *
c *
c -----*
  dimension u(1),dx(1),du(1),w(1),dz(1),dw(1)
c +++
c +++ Constant values
c +++
  rhod=300.    ! Droplet density
  rhog=1.19    ! Fluid density
  ug=0.        ! Fluid velocity in x-direction
  g=9.8       ! Gravity
  wg=-5.0     ! Fluid velocity in z-direction
  eni=0.0004   ! viscosity of the gaseous fluid
c
  ur=abs(sqrt((u(i)-ug)**2+(w(i)-wg)**2))
c
  red=2.*r*ur*rhog/eni
  if (red.le.1000.) then
    cd=24./red*(1.+1./6.*red**(2./3.))
  else
    cd=0.424
  endif
c
  dx(i)=u(i)
  du(i)=1./(rhod*vd)*(-0.5*rhog*(u(i)-ug)*ur*ad*cd)
c
  dz(i)=w(i)
  dw(i)=1./(rhod*vd)*(-0.5*rhog*(w(i)-wg)*ur*ad*cd+
    & (rhog-rhod)*vd*g)
c
  return
  end

```



## References

Leong, M.Y., McDonell, V.G., and Samuelsen, G.S., 2000, "Mixing of an Airblast-Atomized Fuel Spray Injected Into a Crossflow of Air," NASA/CR—2000-210467.

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